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Robust Sequential Detection Using Arrays

A Paper Presented at the 96th Meeting of the Acoustical Society of America

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16 January 1979

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PREFACE

This technical document was prepared under Project No. A-750-28, "Adaptive Nonparametric Sequential Detection," Principal Investigator, Roger F. Dwyer (Code 333).

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26. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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Quantiles

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ROBUST SEQUENTIAL DETECTION USING ARRAYS

INTRODUCTION

I will be discussing a procedure for designing a robust sequential detector using an array of M Hydrophones. The technique has the advantage of not requiring knowledge of the noise distribution. However, a set of quantiles and related functions of the noise distribution are assumed known. In applications, the quantiles and related functions are usually estimated by stochastic approximation techniques. Initially, I will present a detector structure which was derived from the loglikelihood ratio assuming independent samples and discuss its advantages in nongaussian noise. Then I will discuss a procedure to maintain fixed performance levels under dependent sampling.

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ARRAY ROBUST SEQUENTIAL DETECTOR

$$\Lambda(\vec{x}) = \sum_{i=1}^{n} \left[\left(\sigma_{sd}^{2} / 2 \right) \sum_{J=1}^{M} \sum_{K=1}^{m} b_{KJ} n_{KiJ} - B_{I} \right.$$

$$+ \sigma_{sd}^{2} \sum_{J=1}^{M-1} \sum_{K=J+1}^{M} \sum_{K} \sum_{P} b_{KJ} b_{PQ} n_{KiJ} n_{PiA} - B_{J} \right]$$

n - Time to Detection (Random Variable)

M - No. of Hydrophones

B, and B₂ are Constants

52 - Designed Signal Power

 $\sum_{k=1}^{m} b_{k,j}^{'} n_{k,j} - Individual Channel Output$ $\sum_{k=1}^{m} \sum_{p=1}^{m} b_{k,j} b_{pg} n_{k,j} n_{p,g} - Cross Channel Output$

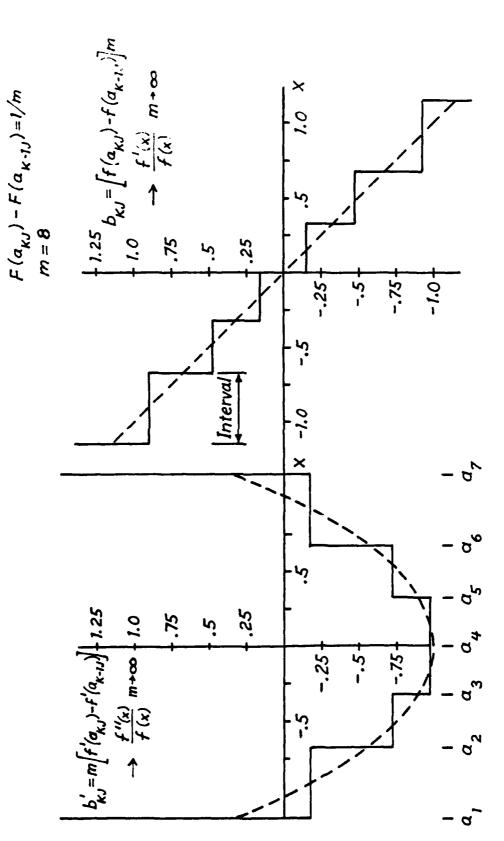
Vu-Graph 1

For a stochastic signal emanating from a farfield point source, the detector structure is composed of two parts. The first term to the right represents the individual hydrophone or channel outputs which gives a non-directional detection capability. The third term represents the cross channel or directional component.

- Lower-case n represents the time to a decision which is a random variable.
- 2. Upper-case M gives the number of hydrophones.
- 3. Upper-case B-sub-one and B-sub-two are constants chosen to ensure the desired false-alarm and false-dismissal probabilities.
- 4. Sigma-squared-sub-sd represents the designed or expected signal power.
- 5. The individual channel outputs are given by the appropriate scores or weights -- lower-case b-prime sub kj -- which are functions of the noise distribution.
- 6. The cross channel outputs are also given by another set of scores -- lower-case b sub kj.

The next vu-graph will indicate how the scores and quantiles are chosen.

QUANTILES AND SCORES FOR GAUSSIAN NOISE

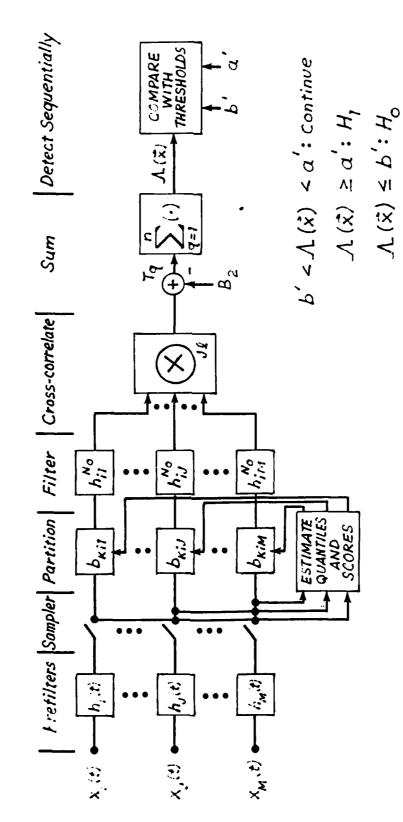


The individual channel scores are shown at the left for zero mean unit variance gaussian noise and assuming the signal is small. The lower-case a's at the bottom represent the quantile locations for an eight interval sequential detector. If the data - represented by X along the horizontal axis - fall into any mutually exclusive interval partitioned by the quantiles, then the output is given by the solid lines along the vertical axis. The scores are given by the difference between the slope of the density function at the quantile locations times the number of intervals - lower-case m. As lower-case m approaches infinity, the scores approach the optimum weighting function for the noise distribution.

The cross channel scores are shown at the right where the same quantiles are used, but the scores are given by the difference between the density function at the quantile locations times the number of intervals. Again, as the number of intervals approach infinity, the scores approach the optimum weighting function, which in general represents a nonlinearity.

If the number of hydrophones in the array is large, usually the individually hydrophone outputs can be neglected compared to the cross-channel outputs, although for some noise distributions this may not be the case. In the following analysis, I will only consider the cross-channel outputs; however, the general case can be developed in an analogous manner.

STRUCTURE OF ROBUST SEQUENTIAL DETECTOR



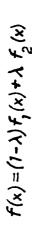
Vu-Graph 3

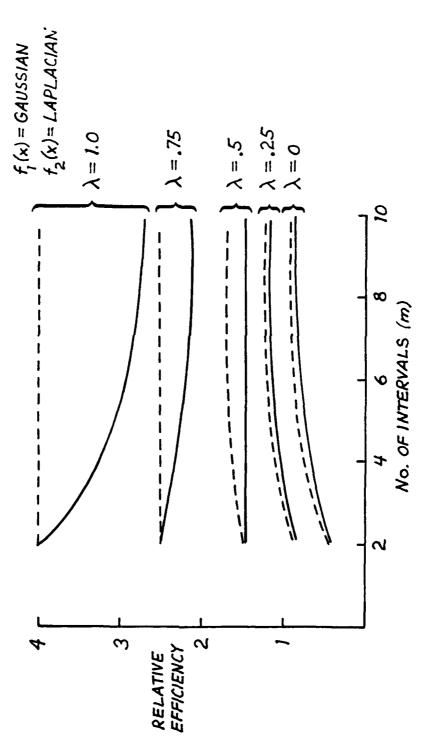
The structure of interest is composed of M Hydrophones, whose spatial correlation between noise-only inputs at different sensors is zero. The signal is assumed to be small and arrives from a specific direction. The delays needed to steer the array have been neglected from the diagram. Each sensor output consists of signal-plus-noise, which is prefiltered and sampled. We assume that the data can be sampled at any rate so that the samples need not be presumed statistically independent. However, it will be assumed that the quantiles and scores for each hydrophone are estimated from an independent identically distributed noise-only learning sample. For weak signals, the quantiles and scores can be estimated under signal plus noise conditions. Once the quantiles and scores are estimated, they are used to construct the m-interval sub-optimum nonlinearity. The outputs are then filtered. The filter can be as simple as a summing operation or as complex as a FFT. N-sub-0 is usually greater than the coherence time of the signal and noise.

The filtered output is cross-correlated and summed over all hydrophone pairs. The constant B-sub-2 is subtracted and the output is further integrated over time, and lambda is compared with the thresholds a-prime and b-prime, which are derived from a generalized version of Wald's Fundamental Identity.

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RELATIVE EFFICIENCY IN ADDITIVE MIXTURE OF GAUSSIAN AND LAPLACIAN NOISE

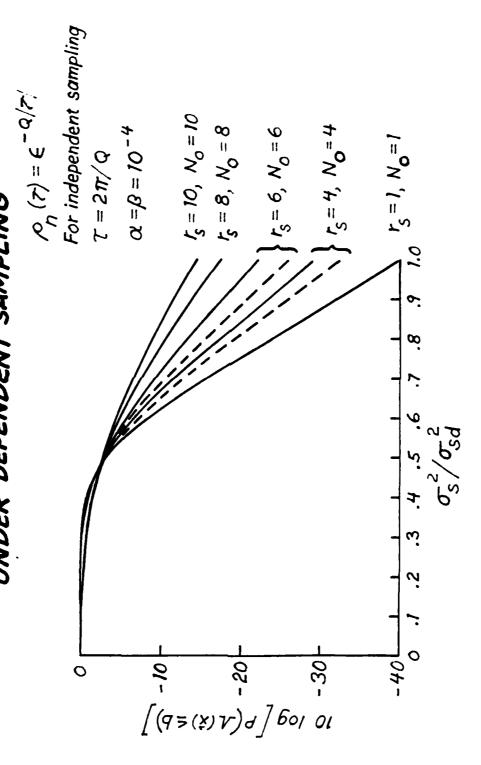




The general form of the robust sequential detector estimates scores as well as quantiles to optimize performance under changing noise distributions. In the simplest implementation, the scores are fixed and the quantiles are estimated, based on equiprobable partitioning. For any arbitrary choice of scores, the thresholds a-prime and b-prime must be adjusted to maintain the desired performance levels.

The relative efficiency of the robust sequential detector compared to a linear sequential detector, based on the averaged time to a decision for identical performance levels, is shown. The linear sequential detector is equivalent to a conventional beamformer followed by a sequential detector. The noise is an additive mixture of gaussian and laplacian (or impulsive) noise. The broken curve shows the performance for the optimum scores under the noise distribution and the solid curve gives the performance for fixed (non-adaptive) scores. For gaussian noise (lambda equal to zero), the relative efficiency increases as the number of intervals increase for both sets of scores. As the noise becomes laplacian (lambda approaches one), the relative efficiency falls off for the fixed non-adaptive scores compared to the optimum scores. However, both curves show improved performance in non-gaussian noise. The performance could have been measured in terms of array gain with the same relative results. Detectors or beamformers, which are matched to the noise have, in general, improved performance or increased array gain compared with detectors or beamformers designed assuming gaussian noise.

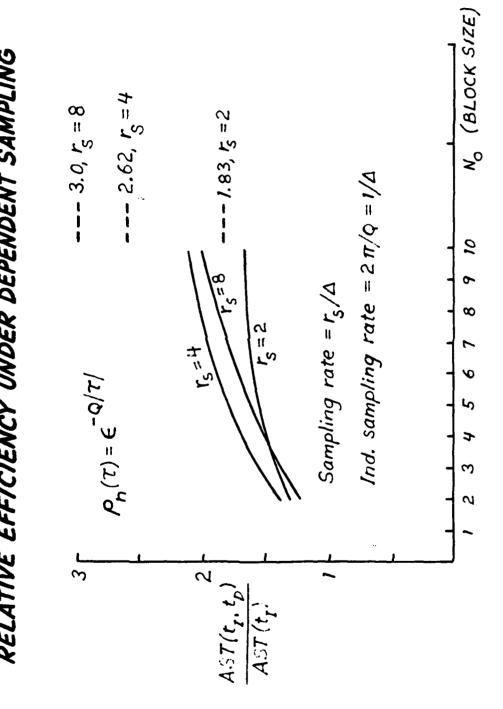
CHANGE IN OPERATING CHARACTERISTIC FUNCTION UNDER DEPENDENT SAMPLING



Vu-Graph 5

Initially, it was assumed that the output of the prefilters were sampled at the nyquist rate and therefore the samples were assumed statistically independent. The assumption was needed to derive the smallsignal structure. If the sampling rate is increased beyond the nyquist rate, the partition output samples will be dependent. As the sampling rate increases, it will no longer be possible to maintain fixed performance levels without adjusting the thresholds. The figure shows the change in the operating characteristic function as the sampling rate increases for the number of intervals approaching infinity. The vertical axis represents the probability of accepting the noise-only condition in dB and the horizontal axis gives the corresponding signal level. The prefilters were assumed to be lowpass filters and the noise gaussian. RHO-sub-N of TAU represents the normalized correlation function of the noise and Q its bandwidth. The false alarm rate alpha and the false dismissal rate beta were nominally set at ten-to-the-minus-4. r-sub-s represents the increase in the sampling rate over the nyquist rate and N-sub-O gives the number of samples summed before cross-correlating the hydrophone pairs. It was also assumed that the signal and noise had the same normalized correlation function. As the sampling rate increases, both the false alarm and false dismissal rates increase unless the thresholds are adjusted proportionally. The broken curve represents the case when the number of intervals equals two. This case is less sensitive to dependence, however, the thresholds must still be adjusted in order to maintain the desired performance levels.

RELATIVE EFFICIENCY UNDER DEPENDENT SAMPLING



Vu-Graph 6

It can be shown that if the thresholds are adjusted properly, the false alarm and the false dismissal rates can be maintained under dependent sampling. The factor needed to adjust the thresholds from their nominal settings depend upon the spectral shape of the noise, the sampling rate, and the number of samples summed. Once the thresholds have been adjusted, the relative efficiency under dependent sampling can be compared to the relative efficiency under independent sampling. The vertical axis represents the improvement in relative efficiency of the array normalized by the relative efficiency under independent sampling. The normalized correlation function of the noise and signal is the same as in the previous vu-graph. As N-Sub-O increases for a fixed sampling rate, the performance increases. For r-Sub-s equal-to-two, the improvement approaches 1.83 as N-Sub-O approaches infinity. The improvement approaches 3 if the increase in the sampling rate is 8.

-- Next Vu-Graph Please --

THRESHOLD ADJUSTMENT FACTOR

For Independent Samples

$$\Lambda(\dot{x}) \begin{cases} \leq b : H_o \\ \geq \alpha : H_I \end{cases}$$

For Dependent Samples

$$\Lambda(\dot{x}) \left\{ \leq bF : H_o \right\}$$

$$\geq \alpha F : H_f$$

$$F = \left[\sum_{V=-(M_o-1)}^{M_o-1} \left(1 - \frac{|V|}{N_o} \right) R_m^N(v) \right]^2 / \sum_{V=-(M_o-1)}^{M_o-1} \left(1 - \frac{|V|}{N_o} \right) R^S(v)$$

Vu-Graph 7

To summarize, under independent sampling, the array output is compared with two thresholds, a and b. The thresholds are derived from Wald's theory on sequential analysis. If the sampling rate is increased so that the samples are dependent, the thresholds must be adjusted by the factor F, where F is given by the equation below. The factor is basically a function of the noise normalized correlation function at the output of the partition and the signal normalized correlation function. The factor can be implemented as shown by adjusting the thresholds or by designing it into the array processor. In the frequency domain, if N sub O represents the DFT or FFT length, and if both r sub s and N sub O approach infinity while the number of intervals m approach infinity and assuming gaussian noise, then the factor F becomes equivalent to the Eckart filter. Therefore, the factor F generalizes the Eckart filter under non-Gaussian noise.

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